SIMULATION THE CONSTITUTIVE MODEL OF DYNAMIC DAMAGE GROWTH OF ROCK MATERIAL

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ABSTRACT

An important feature of rock under dynamic loading is strain-rate dependence. In dynamic loading, stress can exceed the material static strength, but may not damage the material if the nature of the stress is so instant. A change of three orders of magnitude in strain rate can result a change in the failure stress by approximately one order of magnitude (Grady and Kipp, 1987).

A literature review of around 20 articles related with the efforts to find a formula enabling prediction of the level at which the failure will occur, and the processes taking place in the material in the course of loading and eventually leading to failure, has been performed. In order to fully understand failure mechanism of rock material under dynamic loading, three constitutive models revealed from three different institutions are worth discussed further.

This paper is made to entirely elaborate failure mechanism of a rock material under dynamic loading, and no attempts has been put to demonstrate which one is the most appropriate.

1. INTRODUCTION

Compressive failure strength is one of the widely investigated mechanical properties of materials. It is now fairly well established that fracture of brittle solids under uniaxial compression is due to generation and growth of microcracks from pre-existing defects which eventually coalesce to cause axial splitting. Acoustic emission has shown that microfracturing begins at stress of about one-half the ultimate failure stress, and that the number and the length of microcracks increase as the stress is increased further, causing the sample volume to increase even though the stress state is compressive (Ashby and Hallam, 1986).

Several researchers have developed analytical models to simulate damage evolution of brittle solids loaded in uniaxial compression (Nemat-Naseer and Horii,1982; Horii and Nemat-Nasser,1986). Ashby and Hallam (1986) use an approximate method to analyze the growth of a crack from pre-existing flaws. Nemat-Nasser and Deng (1994) extended static model to include dynamic effect by considering an array of interacting and dynamically growing wing cracks, and they estimated the rate-dependent dynamic damage evolution. Ravichandran and Subhash (1995) developed a micromechanical model applicable to brittle solids subjected to biaxial dynamic compressive loading. The model is based on non-interacting, uniformly distributed sliding microcracks that activate when the stress-intensity factor reaches critical value.

Instead of considering the coupling effect of growing cracks in a complicated stress field, several authors adopted either a damage theory or an energetic approach to analyze the fracture failure of microstructurally heterogeneous solids. Grady and Kipp (1980) developed a model for fracture and fragmentation in rocks. The model focused on the observed rate dependence of rock fracture. They introduced a damage parameter, which is based on growth of cracks and the coupling between the distribution of flaws and the rate of loading.

To understand failure mechanism of rocks under dynamic loading, three constitutive models revealed from three different institutions are discussed. The first model is a continuum damage model, which is developed by the Department of Mining Engineering, Queen's University, Canada (Yang et al., 1996; Liu and Katsabanis, 1997). A model that combines damage evolution theory with dynamic crack growth had been successfully developed by the Mechanical Engineering and Engineering Mechanics Department, Michigan Technology University, USA and acknowledged as the second model. Third model is the work of Hao and the co-workers at Protective Technology Research Center, Nanyang Technological University, Singapore (2002). The simulation result is presented and reviewed.

2. DAMAGE VARIABLE

Damage can be defined as any change in the properties of a material that degrades its performance (Singh, 1993). In continuum damage mechanics, Kachanov (1986) proposes the damage variable D which was defined as the part of an area that is unable to support load due to damage A, relative to the area before damage A_0.

$$D = \frac{A}{A_0} \quad (1)$$

Such that D=0 corresponds to the intact, undamaged rock, D=1 represents complete failure, and intermediate values of D correspond to incomplete fracture.

3. CONSTITUTIVE MODELS

3.1. The First Model

Jaeger and Cook (1979) classified five types of the brittle failure for a rock specimen under different loading conditions (Figure 1). In unconfined compression, irregular longitudinal splitting is observed (Figure 1a). With a moderate amount of confining pressure the irregular behavior of Figure 1a is replaced by a single plane of fracture (Figure 1b), inclined at an angle of less than 45^{o} to the direction of normal load. This is a typical fracture under compressive stress. Its characteristics feature is shear displacement along the surface of the fracture. If the confining pressure is increased so that the material become fully ductile (Figure 1c), a network of shear fractures, accompanied by plastic deformation of the individual crystal appears. Another type of fracture, extensional fracture (Figure 1d), occurs in uniaxial tension. Its characteristic is clean separation with no shear offset between the surfaces. If a slab is compressed between the line loads, Figure 1e, an extensional fracture appears between the loads. Under more complicated stress systems, fractures appear which may regarded as belonging to one or more these types.

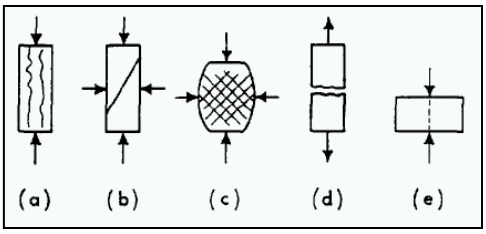

Figure 1. Basic type of rock failure under different loading conditions (after Jaeger and Cook, 1979)

Constitutive equation of the first model was proposed based on the above failure phenomena. Extensional strain criterion (Stacey, 1981) was used to model crack initiation and propagation. Extensional strain is defined as the sum of all the principal tensile strains (logarithmic strain) at a point. In damage model it was assumed that tensile failure would occur (i.e. cracks would develop at a point in rock) if the extensional strain exceeded some critical value. Within a rock flaws and crack exist. Under extensional strain, these flaws or cracks will grow and new ones may be generated. These reduce the strength of rock. The first model was proposed for calculating the crack density increase. It was assumed that:

$$C_d=\alpha(\varepsilon-\varepsilon_c)^{\beta}t \quad (2)$$

where C_d is defined as the total number of cracks per volume; t is time; α and β are material constants; and ε and ε_c are the extensional strain and the critical extensional strain, respectively.

Equation (2) can be written in differential form for programming requirement,

$$dC_d=\alpha\beta(\varepsilon-\varepsilon_c)^{\beta-1}\, d\varepsilon + \alpha(\varepsilon-\varepsilon_c)^{\beta}dt \quad (3)$$

Integrating Equation (3), the crack density is given as,

$$dC_d = \int \alpha\beta(\varepsilon-\varepsilon_c)^{\beta-1}\, t\, \dot{\varepsilon}\, dt + \int \alpha(\varepsilon-\varepsilon_c)^{\beta}\, dt$$
(4)

where $\dot{\varepsilon}$ is the volumetric strain rate expressed by definition using the following formula,

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} \quad (5)$$

The damage parameter was assumed to take the following form,

$$D = 1 - e^{-C_d^{\,2}} \quad (6)$$

where e is the constant base of the natural logarithm.

Kachanov (1986) used Hooke's law to show that the damage constant can be calculated from Young Modulus where E is Young's modulus for undamaged sample and E_d is Young's modulus for damaged sample.

$$E_d = E(1-D) \tag{7}$$

The first simulation was carried out on a hypothetical test of simple uniaxial compression with constant strain rate loading. In this case, the compressive stress is σ_1 and the total extension strain is given by,

$$\varepsilon=\varepsilon_2+\varepsilon_3 \tag{8}$$

with $\varepsilon_2=\varepsilon_3=\nu\sigma_1/E$ (9)

where ν is the Poisson's ratio.

3.2. The Second Model

For the second model, the approach by Nemat-Nasser and Horii (1982) to model the growth of shear sliding-wing cracks is adopted. The model is based on assumption that wing cracks initiate from the most favorably oriented shear cracks and grow along a curved path which eventually turns parallel to the applied load axis in uniaxial compression. Sliding will occur when the local resolved shear stress along the main crcak exceeds the threshold shear friction. With continued increase in applied compressive load, the growth of wing crack will be initiated when the stress-intensity factor K_I of the wing cracks equals the mode I fracture toughness K_{Ic} of the material. The stress-intensity factor K_I driving the growth of wing crack of length l is given as (Ashby and Hallam, 1986),

$$K_I = -\frac{[1-\lambda-\mu(1+\lambda)-4.3\lambda L]}{(1+L)^{3/2}}\left[0.23L+\frac{1}{\sqrt{3(1+L)}}\right]\sigma_1(t)\sqrt{\pi a} \tag{10}$$

where $L=l/a$ is the normalized length of wing crack correspond to grain size $2a$ and $\lambda=\sigma_2/\sigma_1$. In the case of dynamic loading, the mode I dynamic stress-intensity factor was defined in terms of static intensity factor by Freund (1972, 1990) as,

$$K_{Id} = k(\dot{l})K_I \tag{11}$$

The function $k(\dot{l})_I$ represents the inertial effect on crack growth and is given as

$$k(\dot{l}) = \left(1-\frac{\dot{l}}{C_R}\right)\left(1-\frac{\dot{l}}{2C_R}\right)^{-1}{}_I \tag{12}$$

where C_R is the Rayleigh wave speed and can be derived by material parameters, Young's modulus E, density ρ, and Poisson's ratio ν (Meyers, 1994),

$$C_R = \frac{0.862+1.14\nu}{1-\nu}\sqrt{\frac{E}{2(1+\nu)\rho}}{}_I \tag{13}$$

The simplest criterion for the dynamic crack growth is set by assuming that K_{Id} cannot exceed a critical value,

$$K_{Id} = K_{Ic} \tag{14}$$

where K_{Ic} is the static fracture toughness of the material.

Combining Equations (11), (12) and (14), the rate of crack growth becomes,

$$\dot{l} = C_R\frac{K_I-K_{Ic}}{K_I-K_{Ic}/2} \tag{15}$$

Then the crack length is determined incrementally as

$$l^{(i+1)} = l^{(i)} + \dot{l}\Delta t \tag{16}$$

The dynamic damage within a specimen material can simply be expressed by a damage parameter *D(l,t)*, where *l* is the wing crack length and *t* is time. In the following paper, *D(l,t)* is written *D* for simplicity. Same with the first model, *D* can range from 0 to 1 with *D*=0 corresponding to *l*=0 (undamaged) and *D*=1 corresponding to full fragmentation. The intermediate values of *D* represent the material with some level of fracture damage. In an approximate microstructural theory of the elastic properties of fractured rock, Walsh (1965) proposed that *D* can be defined as,

$$D=NV \tag{17}$$

where *N* is the number of flaws per unit volume that are favorable to growth and $V=4\pi l^3/3$ is the spherical region surrounding a flaw of radius *l*, which approximates the stress-relieved volume due to the traction-free boundary of the crack. Since crack length *l* increases with increasing dynamic load, it is also a function of time *t*.

Studies on brittle fracture of rocks and ceramics show that Weibull statistics provides a satisfactory description of the inherent flaw distribution. Hence, the flaw distribution is described by a two parameter Weibull function as,

$$n=k\varepsilon^m \tag{18}$$

where n is the number of flaws which can activate at or below a strain level of ε. The constant *k* and *m* are regarded as material properties characterizing fracture activation.

The damage rate $\dot{D}$ and strain rate $\dot{\varepsilon}$ are presented by stress rate $\dot{\sigma}$ as follows,

$$\dot{D}=\frac{\frac{4}{3}\pi l^3 km\varepsilon^{m-1}\dot{\sigma}/E+4\pi l^2 k\varepsilon^m(1-D)\dot{l}}{1+\frac{4}{3}\pi l^3 n(1-m)} \tag{19}$$

$$\dot{\varepsilon}=\frac{\dot{\sigma}\left(1+\frac{4}{3}\pi n l^3\right)+4Ek\varepsilon^{m+1}\pi l^2(1-D)\dot{l}}{E(1-D)\left[1+\frac{4}{3}\pi n l^3(1-m)\right]} \tag{20}$$

$$\sigma=\sigma(t) \tag{21}$$

where σ =σ(t) is a known loading function.

Knowing the damage rate and strain rate from Equations (19) and (20), strain and damage at any given time can be find by writing the incremental equations as,

$$\varepsilon^{(i+1)}=\varepsilon^{(i)}+\dot{\varepsilon}\Delta t \tag{22}$$

$$D^{(i+1)}=D^{(i)}+\dot{D}\Delta t \tag{23}$$

$$l^{(i+1)}=l^{(i)}+\dot{l}\Delta t \tag{24}$$

For a given stress pulse, the problem can be solved dynamically by first determining the current values of damage rate $\dot{D}$ and strain rate $\dot{\varepsilon}$ from Equations (19) and (20) and then wing-crack growth rate $\dot{l}$ from Equation (15). The corresponding values of the strain *ε*, damage *D*, and crack length *l* can be derived at any given instant time from Equations (22), (23) and (16).

3.3. The Third Model

The third model is the work of Hao and co-workers at Protective Technology Research Center, Nanyang Technological University, Singapore (2002). Having paid attention algorithm of the third model, seems that it was developed originally from the fracture strength model based on inherent flaw concepts of Grady and Kipp (1987). The evolution damage is determined by the number of cracks which activate at the time *t* as follows,

$$D(t) = \int_{tci}^{t} \dot{N}(s)A(t-s)ds \quad (25)$$

where t_{ci} is the time duration needed for fracture to take place, and *A(t-s)* is the stress-relieved area which is determined by a microstructural law for the growth of cracks activated at past time *s*. Assuming the microcracks are penny shaped, it has

$$A(t-s) = \pi C_g^2 (t-s)^2 \quad (26)$$

where C_g is the crack growth velocity, the relation $C_g = 0.38\ UV$ (ultrasonic velocity) is assumed (Robert and Wells, 1954). Noted that the derivation of Equation (26) is based on the assumption that the growth velocity reaches C_g very quickly as soon as a crack activates.

As for the variable *N* in Equation (25), it is the number of microcracks per unit area, and the rate of microcrack activation can be calculated by,

$$\dot{N} = \alpha \langle \varepsilon - \varepsilon_{cr} \rangle^{\beta} \quad (27)$$

In equation above, the angular bracket $\langle . \rangle$ denotes a function defined by $\langle x \rangle = (|x| + x)/2$, and in which α and β are material parameters, and the ε is the strain. Subtituting Equation (27) and (26) into Equation (15) gives

$$D(t) = \alpha \pi C_g^2 \int \langle \varepsilon - \varepsilon_{cr} \rangle^{\beta} (t-s)^2 ds \quad (28)$$

and the normal stress can be obtained using equation as follows,

$$\sigma = (1-D)^2 E\varepsilon \quad (29)$$

4. SIMULATION RESULTS

In order to make comparison between the models, the experimental data of dolomite subjected to uniaxial compression load in a split Hopkinson pressure bar, which was done by Huang et al. (2002), is used within the three models. The various parameters and material properties used in calculations are presented in Table 1. Three damage models are implemented using FORTRAN90.

Table 1. Parameters and material properties of Michigan Dolomite (Subash, 2002)

Parameter	*Value*
Young's Modulus, E, GPa	30.5
Coefficient of friction, μ	0.3
Poisson's ratio, ν	0.2
Bulk material density, ρ, g/mm^3	2.52
Fracture toughness, K_{IC}, $MPam^{1/2}$	0.25
Crack length (grain size), 2a, μm	60
Weibull parameter, k, m^{-3}	1.7×10^{27}
Weibull parameter, m	8
*Critical Strain, ε_c	0.0001
**Ultrasonic Velocity, UV, m/s	2800

Note for Dolomite:
* Stacey (1981)
** Simangunsong (2001)

4.1. The First Model

Since there is no data available for the critical extensional strain, ε_c was taken from the work by Stacey (1981; see Table 1). For simplicity, β is assumed to be one. The value of α was obtained using value from Yang (1996). Figure 2 shows stress-strain relationship as a function of strain rate, the curve almost similar with the typical stress-strain curve from static uniaxial compressive test. It shows a rapid increase in failure stress with increasing strain rate. Figure 3 shows stress and damage as a function of compressive strain for a strain rate 1000/s. It can be seen that the peak stress before strain softening occurs at a damage level of approximately 0.22, this agrees with what was assumed by Grady and Kipp (1987).

4.2. The Second Model

The various parameters and rock properties used in calculation of the second model are presented in Table 1. Simulation results can be seen in Figure 4 and well agree with work of Huang et al. (2002). Failure stress increases proportionally with additional strain rate, but not as big as multiplication of the first model. Simulation result shows that crack growth velocity on a material can exceed its P-wave velocity, which contradicts with the finding of Robert and Wells (1954), in which the maximum crack growth velocity is 0.38 times P-wave velocity. The damage curve reveals that initially the damage is negligible but grows rapidly once a critical damage level is attained. The critical damage level is very small (~0.12).

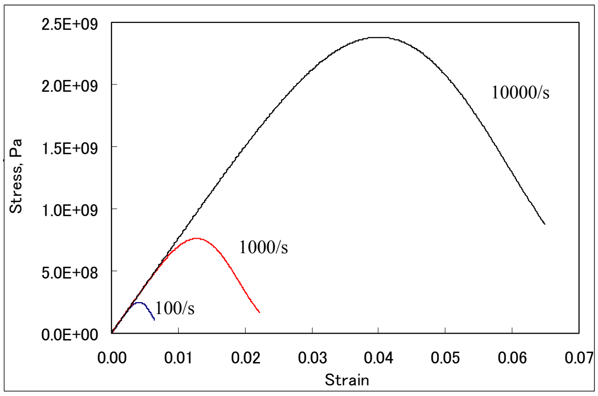

Figure 2. Stress-strain relationship as a function of strain rate (First Model)

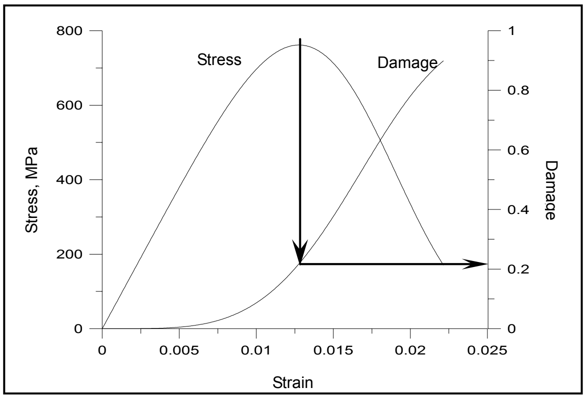

Figure 3. Stress and damage as a function of strain rate of 1000/s (First Model)

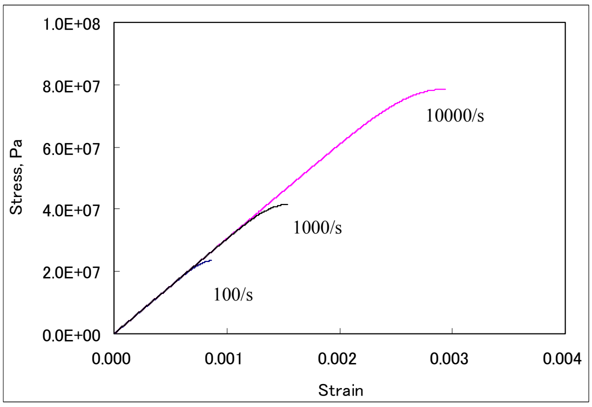

Figure 4. Stress-strain relationship as a function of strain rate (Second Model)

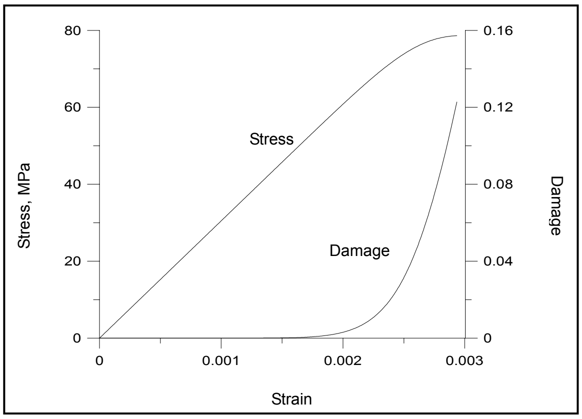

Figure 5. Stress and damage as a function of strain rate of 10000/s (Second Model)

4.3. The Third Model

Using rock properties shown in Table 1 and test data reported by Zhang et al. (2003), the rock parameters in the third model is determined. The parameter β is taken to be 6 so the failure stress is cube root dependent on loading rate, and the crack growth velocity Cg equals to 0.38 times as large as the ultrasonic velocity (Robert and Wells, 1954). The parameter α is taken to be 1.2 x $10^{25}/m^2s$ (Zhang et al., 2003). Simulation results can be seen in Figure 6. Failure stress increases proportionally with additional strain rate. According to the simulation results, the damage value is less then 0.1 when the dynamic compressive stress reach the dynamic failure stress [Figure 7], which is also the minimum damage value for the beginning of fragmentation.

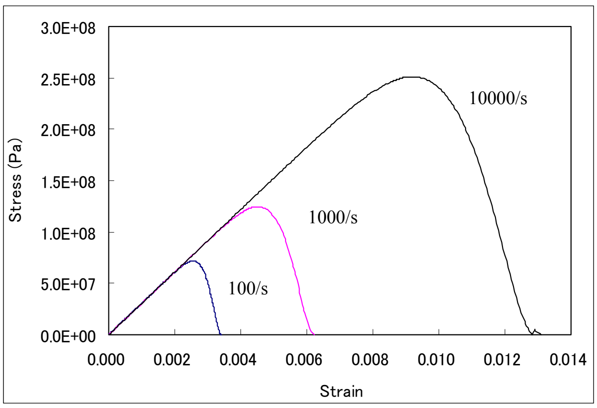

Figure 6. Stress-strain relationship as a function of strain rate (Third Model)

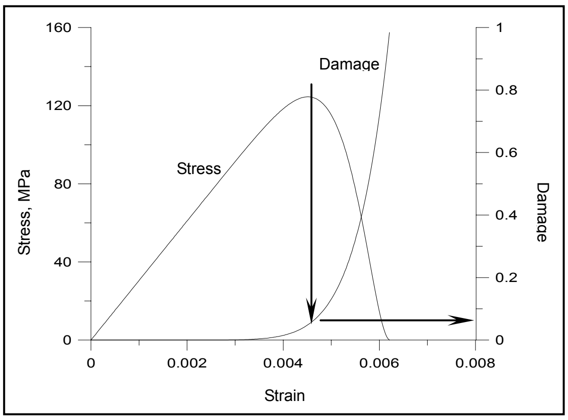

Figure 7. Stress and damage as a function of strain rate of 1000/s (Third Model)

5. SUMMARY

According to the simulation results, all models reveal that failure stress increase proportionally with additional strain rate. Figure 8 shows prediction of failure stress as a function of strain rate for each model. One is different to another on prediction of the level of stress failure. Unfortunately, no attempt can be put to demonstrate which one is the most appropriate since there is no experimental data yet. Nevertheless derivation of three constitutive models presented in this paper clearly shows two important features; first, the behavior of rock under dynamic loading is strain-rate dependence and second, the concept of continuum damage mechanics is based on the fact that fracture always starts from the existing defects.

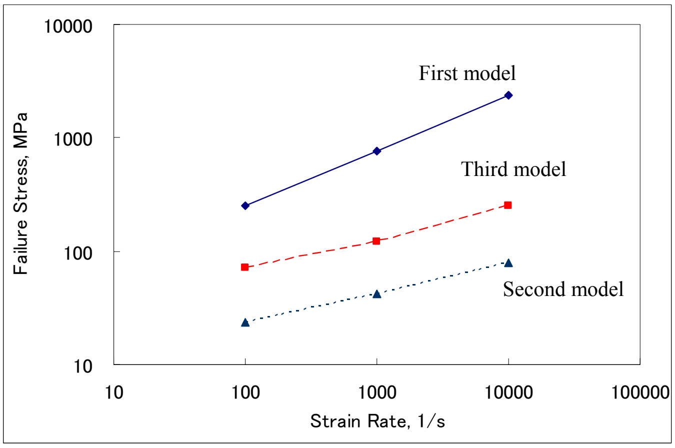

Figure 8. Comparison of predicted failure stress at three different strain rates
(three different models)

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REFERENCES

Ashby, M.F and S.D. Hallam, The failure of brittle solids containing small cracks under compressive stress states, Acta Metall., **34**, pp.497-510 (1986).

Freund, L.B., Crack propagation in an elastic solid subjected to general loading II. Non-uniform rate of extension, J. Mech. Phys. Solids, **20**, pp.141-152 (1972)

Freund, L.B., Dynamic fracture mechanics, Cambridge University Press, New York, NY (1990).

Grady, D.E. and M.E. Kipp, Continuum modeling of explosives fracture in oil shale, Int. J. Rock Mech. Min. Sci. & Geomech. Abstract, **17**, pp.147-157 (1980)

Grady, D.E. and M.E. Kipp, *Dynamic Rock Fragmentation* (in Fracture Mechanics of Rock), Academic Press Inc. (London) Ltd., **10**, pp.1429-475 (1987).

Hao, H., C. Wu, and C.C. Seah, Numerical analysis of blast-induced stress waves in a rock mass with anisotropic continuum damage models, Part 2: Stochastic approach, Rock Mech. Rock Engng, **35**(2), pp.95-108 (2002).

Hao, H., C. Wu, and Y. Zhou, Numerical analysis of blast-induced stress waves in a rock mass with anisotropic continuum damage models, Part 1: Equivalent material properties approach, Rock Mech. Rock Engng, **35**(2), pp.79-94 (2002).

Horii, H. and S. Nemat-Nasser, Brittle failure in compression: splitting, faulting and brittle-ductile transition, Philos. Trans. R. Soc. London 319, pp.337-374 (1986)

Huang, C. and Ghatu Subhash, Influence of lateral confinement on dynamic damage evolution during uniaxial compressive response of brittle solid, Journal of the Mechanics and Physics, ARTICLE IN PRESS (2001).

Huang, C., G. Subhash, and S.J. Vitton, A dynamic growth model for uniaxial compressive response of rock aggregates, Mechanics of Material, **34**, pp.267-277 (2002).

Jaeger, J.C. and N.G.W. Cook, Fundamental of rock mechanics, Third edition, Chapman and Hall, Londong (1979).

Kachanov, L.M., Introduction to damage mechanics, Martinus nijhoff, Dordrect (1986).

Liu, L. and P.D. Katsabanis, Development of a continuum damage model for blasting analysis, Int. J. Rock Mech. Min. Sci., **34**, pp.217-231 (1997).

Meyers, M.A., Dynamic behavior of materials, Wiley, New York (1994).

Nemat-Nasser, S. and H. Deng, Strain-rate effect on brittle failure in compression, Acta Metall. Mater., **42**, pp.1013-1024 (1994).

Nemat-Nasser, S. and H. Horii, Compression induced non planar crack extension with application to splitting, exfoliation, and rockburst, J. Geophysics Res., **87**, pp.6805-6821 (1982)

Ravichandaran, G and G. Subash, A micromechanical model for high strain rate behavior of ceramics, Int. J. Solid. Struct., **32**, pp.2627-2646 (1995).

Robert, D.K. and A.A. Wells, Velocity of brittle fracture, Engineering, **178**, pp.820-821 (1954).

Stacey, T.R., A simple extension strain criterion for fracture of brittle rock, Int. J. Rock Mech. Min. Sci. & Geomech. Abstract, **18**, pp.469-474 (1981).

Walsh, J.B., The effect of cracks on the compressibility of rock, J. Geophys. Res., **70**, pp.381-389 (1965).

Wu, C., H. Hao, J. Zhao, and Y.X. Zhou, Statistical analysis of anisotropic damage of the bukit timah granite, Rock Mech. Rock Engng, **34**(1), pp.23-38 (2001).

Yang, R., W.F. Bawden, and P.D. Katsabanis, A new constitutive model for blast damage, Int. J. Rock Mech. Min. Sci. & Geomech. Abstract, **33**, pp.245-254 (1996).

Zhang, Y., H. Hao, and Y. Lu, Anisotropic dynamic damage and fragmentation of rock materials under explosive loading, International Journal of Engineering Science, **41**, ARTICLE IN PRESS (2003).